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EXPERIMENTAL TESTS OF CPT SYMMETRY AND QUANTUM MECHANICS AT CPLEAR

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Abstract

We review a phenomenological parametrization of an open quantum-mechanical formalism for CPT violation in the neutral kaon system, and constrain the parameters using fits to recent CPLEAR data.

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1 Introduction and Summary

The neutral kaon system provides one of the most sensitive laboratories for testing quantum mechanics at the microscopic level and for testing discrete symmetries [1]. It is the only place where an experimental violation of CP has yet been seen [2], and it provides the strongest constraint on CPT violation, via an upper bound on the difference between the K^0 and the $\overline{K}^0$ masses in the context of quantum mechanics [3]. CPT symmetry is a property of quantum field theory which follows from locality, causality and Lorentz invariance [4]. It is therefore particulary important to look for CPT violation, which, if observed, would require us to revise one or more of these fundamental principles. In particular, the possibility of CPT violation has been raised in the context of quantum gravity [5], as a result of a possible modification of conventional quantum field theory.

A framework for analyzing this possibility is provided by the formulation [6] of open quantum-mechanical systems which are coupled to an unobserved environment. This would induce a loss of quantum coherence in the observed system, which should be described by a density matrix ρ that obeys a modified quantum Liouville equation

$$\dot{\rho} = i[\rho, H] + \not{\delta} H \rho \tag{1}$$

where the extra term $\propto \not{\delta} H$ may be conjectured to arise from quantum-gravitational effects and have a magnitude which is at most $\mathcal{O}(m_K^2/M_{Pl})$, where $M_{Pl} = 1.2 \times 10^{19}\, GeV$ is the gravitational mass scale obtained from Newton's constant: $M_{Pl} = G_N^{-\frac{1}{2}}$. An equation of the form (1) is supported by one interpretation of string theory [7], but could have more general applicability.

In the case of the neutral kaon system, the open-system equation (1) introduces [6] three CPT-violating parameters α, β, γ if energy and strangeness conservation are assumed, in addition to the CPT-violating parameters $\delta m = m_{K^0} - m_{\overline{K}^0}$ and $\delta\Gamma = \Gamma_{K^0} - \Gamma_{\overline{K}^0}$ that can be discussed in the conventional quantum-mechanical framework. Here we use recent CPLEAR data on tagged kaon decays[8, 9] into 2π final states, together with information on $|\eta_{+-}|$ and the semileptonic decay asymmetry, to obtain bounds on these CPT-violating parameters. A more detailed account appears elsewhere [10, 11].

2 Formalism and relevant observables

In this section we first review aspects of the modifications (1) of quantum mechanics believed to be induced by quantum gravity [6], as argued specifically in the context of a non-critical string analysis [7]. This provides a specific form for the modification (1) of the quantum Liouville equation for the temporal evolution of the density matrix of observable matter [7]

$$\frac{\partial}{\partial t}\rho = i[\rho, H] + \delta \not{H} \rho \qquad ; \qquad \delta \not{H} \equiv i\dot{g}^i G_{ij}[g^j, \rho] \tag{2}$$

where the the g^i are generic field-theory couplings on the string world sheet, and G_{ij} is a metric on the space of these couplings. The extra term $\delta \not{H}$ in (2) is such that the time evolution has the following basic properties:

(i) The total probability is *conserved* in time

$$\frac{\partial}{\partial t}\mathrm{Tr}\,\rho = 0 \tag{3}$$

(ii) The energy E is *conserved on the average*

$$\frac{\partial}{\partial t}\mathrm{Tr}\,(E\rho) = 0 \tag{4}$$

as a result of the *renormalizability* of the world-sheet σ-model describing string propagation in a string space-time foam background.

(iii) The von Neumann entropy $S \equiv -k_B \mathrm{Tr}\,\rho \ln \rho$ increases *monotonically with time*

$$\frac{\partial}{\partial t} S \geq 0 \tag{5}$$

which vanishes only if one restricts one's attention to critical (conformal) strings, in which case there is no arrow of time [7]. However, we argue that quantum fluctuations in the background space time should be treated by including non-critical (Liouville) strings [12, 13], in which case (5) becomes a strict inequality. This latter property also implies that the statistical entropy $S_{\mathrm{st}} \equiv \mathrm{Tr}\,\rho^2$ is also monotonically increasing with time, pure states evolve into mixed ones and there is an arrow of time in this picture [7].

(iv) Correspondingly, the superscattering matrix $\not{S}$, which is defined by its action on asymptotic density matrices

$$\rho_{out} = \not{S}\rho_{in} \tag{6}$$

cannot be factorised into the usual product of the Heisenberg scattering matrix and its hermitian conjugate

$$\not{S} \neq SS^\dagger \qquad ; \qquad S = e^{-iHt} \tag{7}$$

with H the Hamiltonian operator of the system. In particular this property implies that $\not{S}$ has no inverse, which is also expected from the property (iii).

(v) The absence of an inverse for $\not{S}$ implies that *strong* CPT invariance of the low-energy subsystem is lost, according to the general analysis of [5, 7].

It should be stressed that, although for the purposes of the present work we keep the microscopic origin of the quantum-mechanics-violating terms unspecified, it is only in the non-critical string model of Ref. [7] - and the associated approach to the nature of time - that a concrete microscopic model guaranteeing the properties (i)-(v) has so far emerged naturally. Moreover, it is worth pointing out that within the non-critical-string framework, we expect that the string σ-model coordinates g^i obey renormalization-group equations of the general form

$$\dot{g}^i = \beta^i M_{Pl} \qquad : \qquad |\beta^i| = \mathcal{O}\left(\frac{E^2}{M_{Pl}^2}\right) \tag{8}$$

where the dot denotes differentiation with respect to the target time, measured in string (M_{Pl}^{-1}) units, and E is a typical energy scale in the observable matter system. Since G_{ij} and g^i are themselves dimensionless numbers of order unity, we expect that

$$|\delta \not{H}| = \mathcal{O}\left(\frac{E^2}{M_{Pl}}\right) \tag{9}$$

in general. However, it should be emphasized that there are expected to be system-dependent numerical factors that depend on the underlying string model, and that $|\delta \not{H}|$ might be suppressed by further (E/M_{Pl})-dependent factors, or even vanish. Nevertheless, (9) gives us an order of magnitude to aim for in the neutral kaon system, namely $\mathcal{O}((\Lambda_{\rm QCD} \text{ or } m_s)^2/M_{Pl}) \sim 10^{-19}\, GeV$.

In the formalism of Ref. [6], the extra (non-Hamiltonian) term in the Liouville equation for ρ can be parametrized by a 4×4 matrix $\delta \not{H}_{\alpha\beta}$, where the indices $\alpha, \beta, \ldots$ enumerate the Hermitian σ-matrices $\sigma_{0,1,2,3}$, which we represent in the $K_{1,2}$ basis. We refer the reader to the literature [6, 14] for details of this description, noting here the following forms for the neutral kaon Hamiltonian

$$H = \begin{pmatrix} M - \frac{i}{2}\Gamma - {\rm Re}M_{12} + \frac{i}{2}{\rm Re}\Gamma_{12} & \frac{1}{2}\delta m - \frac{i}{4}\delta\Gamma - i{\rm Im}M_{12} - \frac{1}{2}{\rm Im}\Gamma_{12} \\ \frac{1}{2}\delta m - \frac{i}{4}\delta\Gamma + i{\rm Im}M_{12} - \frac{1}{2}{\rm Im}\Gamma_{12} & M - \frac{i}{2}\Gamma + {\rm Re}M_{12} - \frac{i}{2}{\rm Re}\Gamma_{12} \end{pmatrix} \tag{10}$$

in the $K_{1,2}$ basis, or

$$H_{\alpha\beta} = \begin{pmatrix} -\Gamma & -\frac{1}{2}\delta\Gamma & -{\rm Im}\Gamma_{12} & -{\rm Re}\Gamma_{12} \\ -\frac{1}{2}\delta\Gamma & -\Gamma & -2{\rm Re}M_{12} & -2{\rm Im}M_{12} \\ -{\rm Im}\Gamma_{12} & 2{\rm Re}M_{12} & -\Gamma & -\delta m \\ -{\rm Re}\Gamma_{12} & -2{\rm Im}M_{12} & \delta m & -\Gamma \end{pmatrix} \tag{11}$$

in the σ-matrix basis. As discussed in Ref. [6], we assume that the dominant violations of quantum mechanics conserve strangeness, so that $\delta \not{H}_{1\beta} = 0$, and that $\delta \not{H}_{0\beta} = 0$ so as to conserve probability. Since $\delta \not{H}_{\alpha\beta}$ is a symmetric matrix, it follows that also $\delta \not{H}_{\alpha 0} = \delta \not{H}_{\alpha 1} = 0$. Thus, we arrive at the general parametrization

$$\delta \not{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix} \tag{12}$$

where, as a result of the positivity of the hermitian density matrix ρ [6]

$$\alpha, \gamma > 0, \qquad \alpha\gamma > \beta^2 \ . \tag{13}$$

We recall [14] that the CPT transformation can be expressed as a linear combination of $\sigma_{2,3}$ in the $K_{1,2}$ basis : CPT $= \sigma_3 \cos\theta + \sigma_2 \sin\theta$ for some choice of phase θ. It is apparent that none of the non-zero terms $\propto \alpha, \beta, \gamma$ in $\delta \not{H}_{\alpha\beta}$ (12) commutes with the CPT transformation. In other words, each of the three parameters α, β, γ violates CPT, leading to a richer phenomenology than in conventional quantum mechanics. This is because the symmetric $\delta \not{H}$ matrix has three parameters in its bottom right-hand 2×2 submatrix, whereas the H matrix appearing in the time evolution within quantum mechanics [15] has only one complex CPT-violating parameter δ,

$$\delta = -\tfrac{1}{2} \frac{\frac{1}{2}\delta\Gamma + i\delta m}{\frac{1}{2}|\Delta\Gamma| + i\Delta m} \tag{14}$$

where δm and $\delta\Gamma$ violate CPT, but do not induce any mixing in the time evolution of pure state vectors[15, 14]. The parameters $\Delta m = M_L - M_S$ and $|\Delta\Gamma| = \Gamma_S - \Gamma_L$ are the usual differences between mass and decay widths, respectively, of K_L and K_S states. For more details we refer the reader to the literature [14]. The above results imply that the experimental constraints [3] on CPT violation have to be rethought. There are essential differences between quantum-mechanical CPT violation[15] and the non-quantum-mechanical CPT violation induced by the effective parameters α, β, γ [6, 14].

Useful observables are associated with the decays of neutral kaons to 2π or 3π final states, or semileptonic decays to $\pi l \nu$. In the density-matrix formalism introduced above, their values are given [6] by expressions of the form

$$< O_i >= Tr(O_i \rho) \tag{15}$$

where the observables O_i are represented by 2×2 hermitian matrices. For future use, we give their expressions in the $K_{1,2}$ basis

$$O_{2\pi} \ = \ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \qquad O_{3\pi} \propto \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{16}$$

$$O_{\pi^- l^+ \nu} \ = \ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \qquad O_{\pi^+ l^- \bar{\nu}} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \tag{17}$$

which constitute a complete hermitian set. In this formalism, pure K^0 or $\overline{K}^0$ states, such as the ones used as initial conditions in the CPLEAR experiment [8, 9] are described by the following density matrices

$$\rho_{K^0} = \tfrac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \qquad \rho_{\overline{K}^0} = \tfrac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \tag{18}$$

We note the similarity of the above density matrices (18) to the semileptonic decay observables in (17), which is due to the strange quark (s) content of the kaon $K^0 \ni \bar{s} \to \bar{u} l^+ \bar{\nu}, \overline{K}^0 \ni s \to u l^- \nu$, and our assumption of the validity of the $\Delta S = \Delta Q$ rule.

Below, we shall apply the above formalism to compute[10] the time evolution of certain experimentally-observed quantities that are of relevance to the CPLEAR experiment[8, 9]. These are asymmetries associated with decays of an initial K^0 beam as compared to corresponding decays of an initial $\overline{K}^0$ beam

$$A(t) = \frac{R(\overline{K}^0{}_{t=0} \to \bar{f}) - R(K^0{}_{t=0} \to f)}{R(\overline{K}^0{}_{t=0} \to \bar{f}) + R(K^0{}_{t=0} \to f)} \ , \tag{19}$$

where $R(K^0 \to f) \equiv \mathrm{Tr}\,[O_f \rho(t)]$, denotes the decay rate into the final state f, given that one starts from a pure K^0 at $t = 0$, whose density matrix is given in (18), and $R(\overline{K}^0 \to \bar{f}) \equiv \mathrm{Tr}\,[O_{\bar{f}} \bar{\rho}(t)]$ denotes the decay rate into the conjugate state $\bar{f}$, given that one starts from a pure $\overline{K}^0$ at $t = 0$.

To determine the temporal evolution of the above observables, which is crucial for experimental fits, it is necessary to know the equations of motion for the components of ρ in the $K_{1,2}$ basis. These are [6, 14][1]

$$\dot{\rho}_{11} = -\Gamma_L \rho_{11} + \gamma \rho_{22} - 2\mathrm{Re}\,[(\mathrm{Im}M_{12} - i\beta)\rho_{12}] \,, \tag{20}$$

$$\dot{\rho}_{12} = -(\Gamma + i\Delta m)\rho_{12} - 2i\alpha \mathrm{Im}\,\rho_{12} + (\mathrm{Im}M_{12} - i\beta)(\rho_{11} - \rho_{22}) \,, \tag{21}$$

$$\dot{\rho}_{22} = -\Gamma_S \rho_{22} + \gamma \rho_{11} + 2\mathrm{Re}\,[(\mathrm{Im}M_{12} - i\beta)\rho_{12}] \,, \tag{22}$$

where ρ represents either $\Delta\rho$ or $\Sigma\rho$, defined by the initial conditions

$$\Delta\rho(0) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \ , \qquad \Sigma\rho(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ . \tag{23}$$

corresponding to the difference between (sum of) initially-pure K^0 and $\overline{K}^0$ states. In these equations, $\Gamma_L = (5.17 \times 10^{-8}\,\mathrm{s})^{-1}$ and $\Gamma_S = (0.8922 \times 10^{-10}\,\mathrm{s})^{-1}$ are the inverse K_L and K_S lifetimes, $\Gamma \equiv (\Gamma_S + \Gamma_L)/2$, $|\Delta\Gamma| \equiv \Gamma_S - \Gamma_L = (7.364 \pm 0.016) \times 10^{-15}\,GeV$, and $\Delta m = 530.0 \times 10^7\,\mathrm{s}^{-1} = 3.489 \times 10^{-15}\,GeV$ is the $K_L - K_S$ mass difference. Also, the CP impurity parameter ϵ is given by

$$\epsilon = \frac{\mathrm{Im}M_{12}}{\frac{1}{2}|\Delta\Gamma| + i\Delta m} \ , \tag{24}$$

which leads to the relations

$$\mathrm{Im}\,M_{12} = \frac{1}{2}\frac{|\Delta\Gamma||\epsilon|}{\cos\phi}, \quad \epsilon = |\epsilon| e^{-i\phi} \quad : \quad \tan\phi = \frac{\Delta m}{\frac{1}{2}|\Delta\Gamma|}, \tag{25}$$

with $|\epsilon| \approx 2.2 \times 10^{-3}$ and $\phi \approx 45^\circ$ the "superweak" phase.

These equations are to be compared with the corresponding quantum-mechanical equations of Ref. [15, 14]. The parameters δm and β play similar roles, although they appear with different relative signs in different places, because of the symmetry

[1] Since we neglect ϵ' effects and assume the validity of the $\Delta S = \Delta Q$ rule, in what follows we also consistently neglect $\mathrm{Im}\,\Gamma_{12}$ [10].

of $\delta \not{H}$ as opposed to the antisymmetry of the quantum-mechanical evolution matrix H. These differences are important for the asymptotic limits of the density matrix, and its impurity. A pure state will remain pure as long as $\mathrm{Tr}\,\rho^2 = (\mathrm{Tr}\,\rho)^2$ [6]. In the case of 2×2 matrices $\mathrm{Tr}\,\rho^2 = (\mathrm{Tr}\,\rho)^2 - 2\det\rho$, and therefore the purity condition is equivalently expressed as $\det\rho = 0$. The existence of the $\delta \not{H}$ term (12) obviously violates this condition, thereby leading to mixed states.

To make a consistent phenomenological study of the various quantities discussed above, it is essential to solve the coupled system of equations (20) to (22) for intermediate times. This requires approximations in order to get analytic results [16]. Writing

$$\rho_{ij}(t) = \rho_{ij}^{(0)}(t) + \rho_{ij}^{(1)}(t) + \rho_{ij}^{(2)}(t) + \cdots \tag{26}$$

where $\rho_{ij}^{(n)}(t)$ is proportional to $\widehat{\alpha}^{p_\alpha}\widehat{\beta}^{p_\beta}\widehat{\gamma}^{p_\gamma}|\epsilon|^{p_\epsilon}$, with $p_\alpha + p_\beta + p_\gamma + p_\epsilon = n$, one can obtain a set of differential equations at each order in perturbation theory.

At order n the differential equations are of the form

$$\frac{d}{dt}\left[e^{At}\rho_{ij}^{(n)}(t)\right] = e^{At}\sum_{i'j'}{}'\rho_{i'j'}^{(n-1)}(t) \tag{27}$$

which can be integrated straightforwardly in terms of the known functions at the $(n-1)$-th order, and the initial condition $\rho_{ij}^{(n)}(0) = 0$, for $n \geq 1$, *i.e.*,

$$\rho_{ij}^{(n)}(t) = e^{-At}\int_0^t dt'\, e^{At'} \sum_{i'j'}{}'\rho_{i'j'}^{(n-1)}(t') \ . \tag{28}$$

Following this straightforward (but tedious) procedure we can obtain the expressions for $\Delta\rho$ at any desired order.

3 Analytical results

We now proceed to give explicit expressions for the temporal evolution of the asymmetries $A_{2\pi}$, $A_{3\pi}$, A_{T}, A_{CPT}, and $A_{\Delta m}$ that are the possible objects of experimental study, in particular by the CPLEAR collaboration [8, 9]. A more detailed account of these results is given in ref. [10][2].

3.1 $A_{2\pi}$

The formula for this asymmetry, as obtained by applying the formalism of section 2, assumes the form

$$A_{2\pi} = \frac{\mathrm{Tr}\,[O_{2\pi}\bar{\rho}(t)] - \mathrm{Tr}\,[O_{2\pi}\rho(t)]}{\mathrm{Tr}\,[O_{2\pi}\bar{\rho}(t)] + \mathrm{Tr}\,[O_{2\pi}\rho(t)]} \equiv \frac{\mathrm{Tr}\,[O_{2\pi}\Delta\rho(t)]}{\mathrm{Tr}\,[O_{2\pi}\sum\rho(t)]} \ , \tag{29}$$

[2]See also ref. [17].

where the observable $O_{2\pi}$ is given in (17), and the $\Delta\rho$ and $\sum\rho$ density matrix elements are given as above (23). The result for the asymmetry, to second order in the small parameters, can be written most concisely as

$$\begin{aligned}A_{2\pi}(t) \quad = \quad & \Big\{ 2|\epsilon|\cos\phi + 4\widehat{\beta}\sin\phi\cos\phi - 8\widehat{\alpha}\sin\phi\cos\phi(|\epsilon|\sin\phi - 2\widehat{\beta}\cos^2\phi) \\ & -2\sqrt{|\epsilon|^2 + 4\widehat{\beta}^2\cos^2\phi}\; e^{\frac{1}{2}(\Gamma_S-\Gamma_L)t}\left[\cos(\Delta mt - \phi - \delta\phi) + \frac{2\widehat{\alpha}}{\tan\phi}X_\alpha\right]\Big\} \\ & / \left\{1 + e^{(\Gamma_S-\Gamma_L)t}\left[\widehat{\gamma} + |\epsilon|^2 - 4\widehat{\beta}^2\cos^2\phi - 4\widehat{\beta}|\epsilon|\sin\phi\right]\right\} \end{aligned} \qquad (30)$$

where $\widehat{\alpha}, \widehat{\beta}, \widehat{\gamma}$ are scaled variables $a/\Delta\Gamma$,$\beta/\Delta\Gamma$, $\gamma/\Delta\Gamma$, and $X_\alpha \equiv \cos\delta\phi\sin(\Delta mt - \phi) - \frac{1}{2}|\Delta\Gamma|t\tan\phi\cos(\Delta mt - \phi - \delta\phi) + \sin\phi\cos(\Delta mt - 2\phi - \delta\phi)$.

The above expression should be compared with the usual case (*i.e.*, $\widehat{\alpha} = \widehat{\beta} = \widehat{\gamma} = 0$)

$$A_{2\pi}(t) = \frac{2|\epsilon|\cos\phi - 2|\epsilon|\, e^{\frac{1}{2}(\Gamma_S-\Gamma_L)t}\,\cos(\Delta mt - \phi)}{1 + e^{(\Gamma_S-\Gamma_L)t}\,|\epsilon|^2} \qquad (31)$$

One can readily see whether CP violation can in fact vanish, its effects mimicked by non-quantum-mechanical CPT violation. Setting $|\epsilon| = 0$ one needs to reproduce the interference pattern and also the denominator. To reproduce the overall coefficient of the interference pattern requires $2\widehat{\beta}\cos\phi = \pm|\epsilon|$. The denominator (neglecting $\widehat{\gamma}$) becomes $-4\widehat{\beta}^2\cos^2\phi \to -|\epsilon|^2$ and we have the wrong sign. Another problem is that $\delta\phi \to -\mathrm{sgn}(\widehat{\beta})\frac{\pi}{2}$ and the interference pattern is shifted significantly. This means that the effects seen in the neutral kaon system, and conventionally interpreted as CP violation, indeed cannot be due to the CPT violation.

3.2 $A_{3\pi}$

Analogously, the formula for the 3π asymmetry, to first order in the small parameters, is given by

$$A_{3\pi}(t) = \frac{\left[2|\epsilon|\cos\phi - 4\widehat{\beta}\sin\phi\cos\phi\right] - 2e^{-\frac{1}{2}(\Gamma_S-\Gamma_L)t}\left[\mathrm{Re}\eta_{3\pi}\cos\Delta mt - \mathrm{Im}\eta_{3\pi}\sin\Delta mt\right]}{1 + \widehat{\gamma} - \widehat{\gamma}e^{-(\Gamma_S-\Gamma_L)t}}, \qquad (32)$$

with

$$\mathrm{Re}\eta_{3\pi} = |\epsilon|\cos\phi - 2\widehat{\beta}\sin\phi\cos\phi, \quad \mathrm{Im}\eta_{3\pi} = |\epsilon|\sin\phi + 2\widehat{\beta}\cos^2\phi\ . \qquad (33)$$

In the CPLEAR experiment, the time-dependent decay asymmetry into $\pi^0\pi^+\pi^-$ is measured [8, 9], and the data is fit to obtain the best values for $\mathrm{Re}\eta_{3\pi}$ and $\mathrm{Im}\eta_{3\pi}$.

3.3 A_{T}

In the CPT-violating case, to first order in the small parameters, one finds the following time-dependent expression for this asymmetry

$$A_{\mathrm{T}} = \frac{4|\epsilon|}{\cos\delta\phi} \left\{ \frac{e^{-\Gamma_L t}\cos(\phi-\delta\phi) + e^{-\Gamma_S t}\cos(\phi+\delta\phi) - 2e^{-\Gamma t}\cos(\Delta m t - \delta\phi)\cos\phi}{e^{-\Gamma_L t}(1+2\widehat{\gamma}) + e^{-\Gamma_S t}(1-2\widehat{\gamma}) - 2e^{-\Gamma t}[\cos\Delta m t + \frac{2\widehat{\alpha}}{\tan\phi}(\sin\Delta m t - \Delta m t\cos\Delta m t)]} \right\} \tag{34}$$

where we have defined

$$\tan\delta\phi = -\frac{2\widehat{\beta}\cos\phi}{|\epsilon|} + \ldots, \tag{35}$$

with the ... denoting higher order corrections[10]. The expression (34) aymptotes to

$$A_{\mathrm{T}} \rightarrow \frac{4|\epsilon|\cos(\phi-\delta\phi)}{\cos\delta\phi(1+2\widehat{\gamma})} = \frac{4|\epsilon|\cos\phi - 8\widehat{\beta}\sin\phi\cos\phi}{1+2\widehat{\gamma}} . \tag{36}$$

3.4 A_{CPT}

Following the discussion in section 2, the formula for this observable, as defined by the CPLEAR Collaboration [9], is given by Eq. (19) with $f = \pi^- l^+ \nu$ and $\bar{f} = \pi^+ l^- \bar{\nu}$. To first order, in both the CPT-conserving and CPT-violating cases, we find

$$A_{\mathrm{CPT}} = 0 . \tag{37}$$

We point out that this result is a quite distinctive signature of the modifications of the quantum mechanics proposed in Ref. [6, 14], since in the case of quantum-mechanical violation of CPT symmetry [15] there is a non-trivial change in A_{CPT}, proportional to the CPT-violating parameters δm and $\delta\Gamma$. Indeed, we obtain [10] the following first-order asymptotic result

$$A_{\mathrm{CPT}}^{\mathrm{QM}} \rightarrow 4\sin\phi\cos\phi\,\widehat{\delta m} + 2\cos^2\phi\,\widehat{\delta\Gamma} , \tag{38}$$

written in terms of the scaled variables. Part of the reason for this difference is the different role played by δm as compared to the β parameter in the formalism of Ref. [6], as discussed in detail in Ref. [14]. In particular, there are important sign differences between the ways that δm and β appear in the two formalisms, that cause the suppression to second order of any quantum-mechanical-violating effects in A_{CPT}, as opposed to the conventional quantum mechanics case.

3.5 $A_{\Delta m}$

Following Ref. [8], one can define $A_{\Delta m}$ as

$$A_{\Delta m} = \frac{R(K^0 \to \pi^+) + R(\overline{K}^0 \to \pi^-) - R(\overline{K}^0 \to \pi^+) - R(K^0 \to \pi^-)}{R(K^0 \to \pi^+) + R(\overline{K}^0 \to \pi^-) + R(\overline{K}^0 \to \pi^+) + R(K^0 \to \pi^-)} \tag{39}$$

in an obvious short-hand notation for the final states of the semileptonic decays, where only the pion content is shown explicitly. In the formalism of section 2, this expression becomes in the quantum-mechanics-violating case to first order

$$A_{\Delta m} = -\frac{2e^{-\Gamma t}\left[\cos \Delta m t + \frac{2\widehat{\alpha}}{\tan\phi}(\sin \Delta m t - \Delta m t \cos \Delta m t)\right]}{e^{-\Gamma_L t}(1 + 2\widehat{\gamma}) + e^{-\Gamma_S t}(1 - 2\widehat{\gamma})} \tag{40}$$

Since $\widehat{\gamma}$ is negligible, this observable provides an *exclusive* test of $\widehat{\alpha}$.

4 Regeneration

Regeneration involves the coherent scattering of a K^0 or $\overline{K}^0$ off a nuclear target, which we assume can be described using the normal framework of quantum field theory and quantum mechanics. Thus we describe it by an effective Hamiltonian which takes the form

$$\Delta H = \begin{pmatrix} T + \overline{T} & T - \overline{T} \\ T - \overline{T} & T + \overline{T} \end{pmatrix} \tag{41}$$

in the $K_{1,2}$ basis, where

$$T = \frac{2\pi N}{m_K}\mathcal{M}\,, \qquad \overline{T} = \frac{2\pi N}{m_K}\overline{\mathcal{M}} \tag{42}$$

with $\mathcal{M} = \langle K^0|A|K^0\rangle$ the forward K^0-nucleus scattering amplitude (and analogously for $\overline{\mathcal{M}}$), and N is the nuclear regenerator density. The regenerator effects ΔH can in principle be included as a contribution to H in the density matrix equation:

$$\partial_t \rho = -i[H, \rho] + i\delta H\!\!\!/\,\rho \tag{43}$$

where $\delta H\!\!\!/$ represents the possible CPT- and QM-violating term. However we note that the regenerator provides an 'environment' that induces an effective CPT violation within quantum mechanics, due to the inequivalent scattering of K^0 and $\overline{K}^0$ by the regenerator material.

It may be adequate as a first approximation to treat the regenerator as very thin, in which case we may use the impulse approximation, and the regenerator changes ρ by an amount

$$\delta\rho = -i[\Delta\mathcal{H}, \rho] \tag{44}$$

where

$$\Delta\mathcal{H} = \int dt \Delta H \tag{45}$$

Writing

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12}^* \\ \rho_{12} & \rho_{22} \end{pmatrix} , \tag{46}$$

in this approximation we obtain

$$\delta\rho = -i\Delta T \begin{pmatrix} 2i\mathrm{Im}\rho_{12} & -\rho_{11}+\rho_{22} \\ \rho_{11}-\rho_{22} & -2i\mathrm{Im}\rho_{12} \end{pmatrix} , \tag{47}$$

where

$$\Delta T \equiv \int dt(T-\overline{T}) . \tag{48}$$

This change in ρ enables the possible CPT- and QM-violating terms in (43) to be probed in a new way, as we now discuss in a special case.

Consider the idealization that the neutral K beam is already in a K_L state :

$$\rho = \rho_L \approx \begin{pmatrix} 1 & \epsilon^* + B^* \\ \epsilon + B & |\epsilon|^2 + C \end{pmatrix} \tag{49}$$

where

$$B = -i2\widehat{\beta}\cos\phi\, e^{-i\phi} \qquad ; \qquad C = \widehat{\gamma} - 4\widehat{\beta}^2\cos^2\phi - 4\widehat{\beta}|\epsilon|\sin\phi \tag{50}$$

Substituting Eqs. (49,50) into Eq. (47), we find that in the joint large-t and impulse approximations

$$\rho + \delta\rho = \begin{pmatrix} 1 + 2\Delta T\mathrm{Im}(\epsilon+B) & \epsilon^* + B^* + i(1-|\epsilon|^2 - C)\Delta T \\ \epsilon + B - i(1-|\epsilon|^2 - C)\Delta T & |\epsilon|^2 + C - 2\Delta T\mathrm{Im}(\epsilon+B) \end{pmatrix} . \tag{51}$$

We see that the usual semileptonic decay asymmetry observable

$$O_{\pi^- l^+ \nu} - O_{\pi^+ l^- \bar{\nu}} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \qquad , \tag{52}$$

which measures $\mathrm{Re}(\epsilon+B)$ in the case without the regenerator, receives no contribution from the regenerator (*i.e.*, ΔT cancels out in the sum of the off-diagonal elements). On the other hand, there is a new contribution to the value of $R_{2\pi} = R(K_L \to 2\pi) \propto \mathrm{Tr}[O_{2\pi}\rho] = \rho_{22}$, namely

$$R_{2\pi} = |\epsilon|^2 + \widehat{\gamma} - 4\widehat{\beta}^2\cos^2\phi - 4\widehat{\beta}|\epsilon|\sin\phi - 2\Delta T\mathrm{Im}(\epsilon+B) . \tag{53}$$

The quantity $\mathrm{Im}(\epsilon + B)$ was not accessible directly to the observable $R_{2\pi}$ in the absence of a regenerator. Theoretically, the phases of ϵ and B (50) are fixed, *i.e.*,

$$\mathrm{Im}(\epsilon+B) = -|\epsilon|\frac{\sin(\phi-\delta\phi)}{\cos\delta\phi} = -|\epsilon|\sin\phi - 2\widehat{\beta}\sin\phi\cos\phi . \tag{54}$$

Nevertheless, this phase prediction should be checked, so the regenerator makes a useful addition to the physics programme.

The above analysis is oversimplified, since the impulse approximation may not be sufficiently precise, and the neutral K beam is not exactly in a K_L state. However, it may serve to illustrate the physics interest of measurements using a regenerator[10]. We note that measurements with different thicknesses of regenerator should have a distinctive dependence $\Delta R_{2\pi} \propto \Delta T$, which is a nice signature. Moreover, the cylindrical geometry of the CPLEAR detector provides such different measurements "for free" at different planar angles.

5 CPLEAR Bounds on CPT-Violating Parameters

5.1 Description of the CPLEAR Experiment

The CPLEAR experiment[8, 19] is designed to determine CP and T violation in the neutral kaon system by measuring time-dependent decay rate asymmetries of CP and T conjugate processes. Initially-pure K^0 and $\overline{K}^0$ states are produced concurrently in the annihilation channels $(p\overline{p})_{\mathrm{rest}} \rightarrow K^0 K^- \pi^+$ and $(p\overline{p})_{\mathrm{rest}} \rightarrow \overline{K}^0 K^+ \pi^-$, each one with a branching ratio of $\approx 0.2\%$. The strangeness of the neutral kaon is tagged by the charge sign of the accompanying kaon. In *Fig. 1* the decay rates for K^0 and $\overline{K}^0$ measured by our experiment are shown separately, demonstrating the CP violation effect.

A detailed description of the experiment can be found elsewhere [20], and only a few important items are mentioned here. The high rate of 200 MeV/c antiprotons ($10^6\,\overline{p}$/s) is delivered by the LEAR machine at CERN. The antiprotons are stopped inside a gaseous hydrogen target of 16 bar pressure. A cylindrical detector is placed inside a solenoid of 1 m radius, 3.6 m length, providing a magnetic field of 0.44 T. The charged tracking system consists of two proportional chambers, six drift chambers and two layers of streamer tubes. Fast kaon identification is provided by a threshold Čerenkov counter sandwiched between two scintillators. An electromagnetic calorimeter made of 18 layers of Pb converters and streamer tubes is used for photon detection and electron identification. An efficient and fast on-line data reduction is achieved with a multi-level trigger system based on custom-made hardwired processors.

Kinematical constraints (energy-momentum conservation, K^0 mass) and geometrical constraints (K^0 flight direction and vertex separation) are used in the analysis to suppress the background from unwanted K^0 decay channels and the background from $p\overline{p}$ annihilation events. In addition they improve the lifetime resolution.

Knowing the initial strangeness of the neutral kaon, we are able to calculate time-dependent CP-violating decay-rate asymmetries:

$$A_{+-}(t) = \frac{N_{\overline{K}^0\rightarrow\pi^+\pi^-}(t) - \alpha N_{K^0\rightarrow\pi^+\pi^-}(t)}{N_{\overline{K}^0\rightarrow\pi^+\pi^-}(t) + \alpha N_{K^0\rightarrow\pi^+\pi^-}(t)} \tag{55}$$

All the acceptances common for K^0 and $\overline{K}^0$ cancel, thus reducing systematic uncertainties. The normalization factor α is proportional to the tagging efficiency of

$\overline{K}^0$ relative to $K^0$and is determined experimentally from the data together with the CP-violation parameters.

5.2 Description of the fit

The results presented here are preliminary, and based on the CPLEAR data collected up to middle of 1994. The analysis based on standard quantum mechanics is presented in [19].

The measured rates $N_{\overline{K}^0\to\pi^+\pi^-}(t)$ and $N_{K^0\to\pi^+\pi^-}(t)$ have been corrected for regeneration. The residual background contribution, mainly from semileptonic decays, is extracted from a Monte Carlo simulation and taken into account in the fit. A systematic study of the effect of lifetime resolution and the regeneration correction is in preparation.

We use the formalism for $K^0 \to \pi^+\pi^-, \overline{K}^0 \to \pi^+\pi^-$ decays presented in section 3.1. In addition we require the following constraints to be fulfilled:

- $|\eta_{+-}|^2 = \hat{\gamma}^2 + |\varepsilon|^2 \dfrac{\cos(\varphi - 2\delta\varphi)}{\cos(\varphi)\cos^2(\delta\varphi)}$,where $|\eta_{+-}|$ is the CP-violation parameter measured in K_L decays to $\pi^+\pi^-$, taken from the Particle Data Group [3], and $\tan(\delta\varphi) = -\dfrac{2\hat{\beta}\cos(\varphi)}{\varepsilon}$,

- $\delta_L = 2\varepsilon \cdot \cos(\varphi) - 4\hat{\beta}\cos(\varphi)\sin(\varphi)$, where δ_L is the semileptonic asymmetry measured in K_L decays, taken from the Particle Data Group [3],

- $\Delta m = (530.02 \pm 1.63) \cdot 10^7 \hbar s^{-1}$, is the average of the experiments listed in ref. [21].

Fitting the CP asymmetry shown in fig. 2 together with the above constraints, we obtain the following result for the CP- and CPT-violation parameters:

$$\begin{aligned} \varepsilon &= (2.257 \pm 0.011) \times 10^{-3} \\ \hat{\alpha} &= (-4.1 \pm 8.5) \times 10^{-3} \\ \hat{\beta} &= (-1.4 \pm 4.6) \times 10^{-5} \\ \hat{\gamma} &= (-0.3 \pm 3.3) \times 10^{-7} \end{aligned} \tag{56}$$

The χ^2 of the fit is $0.8/dof$.

Limits for $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ are calculated by taking into account only the physical region, i.e., by imposing the positivity constraints $\hat{\alpha}, \hat{\gamma} > 0$ and $\hat{\alpha} \cdot \hat{\gamma} > \hat{\beta}^2$. Integrating over the other two parameters we derive the following 90% confidence level upper limits:

$$\begin{aligned} \hat{\alpha} &< 4.2 \times 10^{-3} \\ |\hat{\beta}| &< 2.1 \times 10^{-5} \\ \hat{\gamma} &< 2.9 \times 10^{-7} \end{aligned} \tag{57}$$

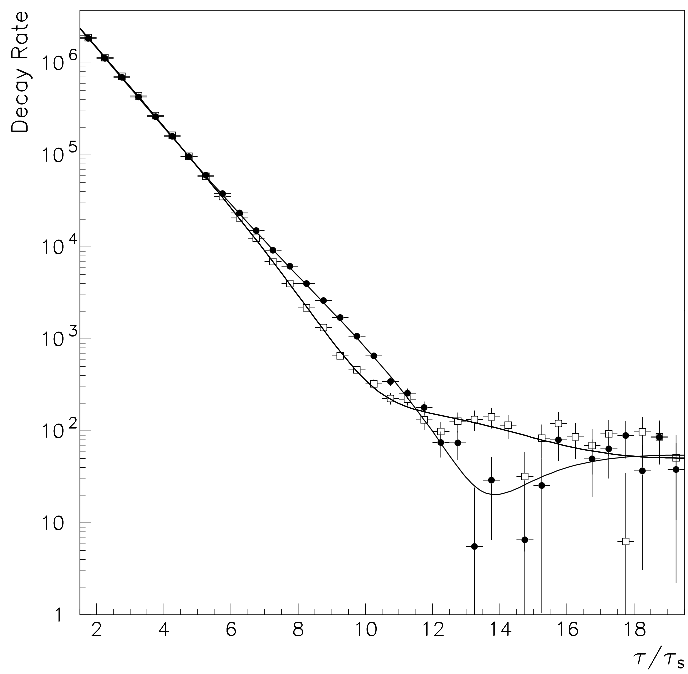

Figure 1: Acceptance-corrected decay rate of $\overline{K}^0$ (filled circles) and K^0 (open squares). Lines are the expected rates when the Particle Data Group [3] values are used.

5.3 Remark on the $K_S - K_L$ mass difference

In all experiments which have measured Δm so far, standard quantum mechanics has been assumed. The CPLEAR collaboration has started to make a common fit of the semileptonic asymmetry $A_{\Delta m}$ and the two-pion asymmetry A_{+-}, keeping Δm as a free parameter. These results will be published soon.

6 Theoretical Comment on two-particle decay correlations

Alternative interesting tests of quantum mechanics and CPT symmetry can be devised by exploiting initial-state correlations due to the production of a pair of neutral kaons

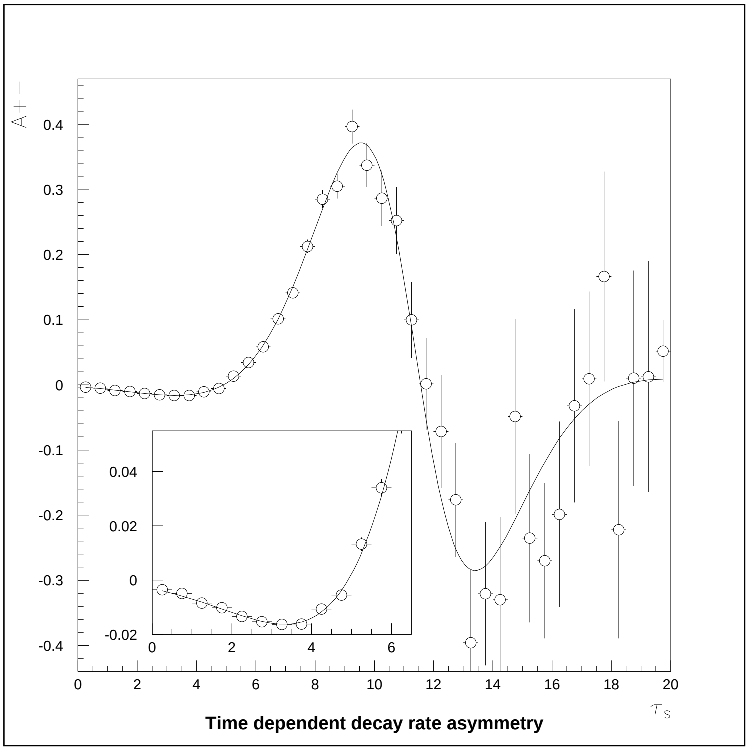

Figure 2: Decay-rate asymmetry as function of the decay eigentime. The line shows the result of the best fit.

in a pure quantum-mechanical state, e.g., via $e^+e^- \to \phi \to K^0\overline{K}^0$. In this case, the initial state may be represented by [18]

$$|\mathbf{k}\ ;\ -\mathbf{k}\rangle = \frac{1}{\sqrt{2}}\left[\left|K^0(\mathbf{k})\ ;\ \overline{K}^0(-\mathbf{k})\right\rangle - \left|\overline{K}^0(\mathbf{k})\ ;\ K^0(-\mathbf{k})\right\rangle\right] \tag{58}$$

At subsequent times $t = t_1$ for particle 1 and $t = t_2$ for particle 2, the joint probability amplitude is given in conventional quantum mechanics by

$$|\mathbf{k},\ t_1\ ;\ -\mathbf{k},\ t_2\rangle \equiv e^{-iH(\mathbf{k})t_1} e^{-iH(-\mathbf{k})t_2}\, |\mathbf{k}\ ;\ -\mathbf{k}\rangle \tag{59}$$

Thus the temporal evolution of the two-particle state is completely determined by the one-particle variables (OPV) contained in H.

Tests of quantum mechanics and CPT symmetry in ϕ decays [22] have recently been discussed [17] in a conjectured extension of the formalism of [6, 14], in which the density matrix of the two-particle system was hypothesized to be described completely in terms of such one-particle variables (OPV): H and (α, β, γ). It was pointed out that this OPV hypothesis had several striking consequences, including apparent violations of energy conservation and angular momentum.

As we have discussed above, the only known theoretical framework in which the EHNS equation has been derived is that of a non-critical string approach to string theory, in which (i) energy is conserved in the mean as a consequence of the renormalizability of the world-sheet σ-model, but (ii) angular momentum is not necessarily conserved, as this is not guaranteed by renormalizability and is not conserved in some toy backgrounds [23], though we cannot exclude the possibility that it may be conserved in some particular backgrounds. Therefore, we are not concerned that [17] find angular momentum non-conservation in their hypothesized OPV approach, but the absence of energy conservation in their approach leads us to the conclusion that irreducible two-particle parameters must be introduced into the evolution of the two-particle density matrix. The appearance of such non-local parameters does not concern us, as the string is intrinsically non-local in target space, and this fact plays a key role in our model calculations of contributions to $\delta \not{H}$. The justification and parametrization of such irreducible two-particle effects goes beyond the scope of these talks, and we plan to study this subject in more detail in due course.

7 Conclusions

We have discussed in these talks approximate expressions for a complete set of neutral kaon decay observables $(\pi\pi, 3\pi, \pi^{\pm}l^{\mp}\nu)$, which can be used to constrain the parameters α, β, γ characterising CPT violation in a formalism motivated by ideas about quantum gravity and string theory, that incorporates a possible microscopic loss of quantum coherence by treating the neutral kaon as an open quantum-mechanical system [6, 7, 14, 10]. Detailed fits to recent CPLEAR experimental data on 2π decays have been reported based on our formulae[11]. These may be used to obtain indicative upper bounds

$$|\alpha| \lesssim 3.1 \times 10^{-17}\, GeV, \; |\beta| \lesssim 1.5 \times 10^{-19}\, GeV, \; |\gamma| \lesssim 2.1 \times 10^{-21}\, GeV \qquad (60)$$

which are comparable with the order of magnitude $\sim 10^{-19}\, GeV$ which theory indicates[7] might be attained by such CPT- and quantum-mechanics-violating parameters.

We have not presented explicit expressions for the case where the deviation $\epsilon'/\epsilon \lesssim 10^{-3}$ from pure superweak CP violation is non-negligible, but our methods can easily be extended to this case. They can also be used to obtain more complete expressions for experiments with a regenerator, if desired. Details of the extension of the formalism of ref. [6] to correlated $K^0\overline{K}^0$ systems produced in ϕ decay, as at DAϕNE [22], involves the introduction of two-particle variables, which lies beyond the scope of this paper.

We close by reiterating that the neutral kaon system is the best microscopic laboratory for testing quantum mechanics and CPT symmetry. We believe that violations of these two fundamental principles, if present at all, are likely to be linked, and have proposed a formalism that can be used to explore systematically this hypothesis, which is motivated by ideas about quantum gravity and string theory. Our understanding of these difficult issues is so incomplete that we cannot calculate the sensitivity which would be required to reveal modifications of quantum mechanics or a violation of CPT. Hence we cannot promise success in any experimental search for such phenomena. However, we believe that both the theoretical and experimental communities should be open to their possible appearance.

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